Math reasoning

Odd & even number operations

1. *even ± even = even*
2. *even ± odd = odd*
3. *odd ± odd = even*
4. *even · even = even*
5. *even · odd = even*
6. *odd · odd = odd*

Number facts to memorize

- Times tables up to 15×15
- Perfect squares up to 20^2
- Perfect cubes up to 10^3
- Powers of base 2 up to 2^{10}
- Powers of base 3 up to 3^5
- Powers of base 3 up to 4^5
- Powers of base 3 up to 5^4
- Prime numbers less than 20 (don't forget 2!)

Positive & negative number operations

1. positive · or ÷ positive = positive
2. positive · or ÷ negative = negative
3. negative · or ÷ negative = positive

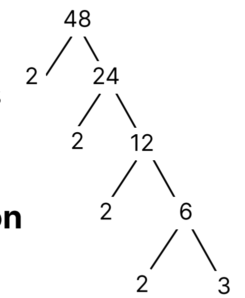
Probability

Probability = desired outcomes/total outcomes

- Probability is shown as fraction or decimal **not** a percent (unless percent is indicated)
- A 100% probability, therefore =**1.**

Strategy insight

The factor tree is your friend! Here's an example:



Prime factorization

$2^4 \times 3$

To quickly check whether a number is divisible by 3, add the number's digits. If the sum of the digits is divisible by 3, then the number itself is divisible by 3. The trick also works for 9!

Average (mean) formulas

Average = sum/number

Also, importantly: **Sum = average x number**

Percent formulas

- Percent = part/whole x 100
- Percent change = (new-old)/old x 100

- To find 10% of a number, drop a zero from the number (or move the decimal one to the left)
- To find 1% of a number, drop two zeroes from the number (or move the decimal two to the left)
- With percents, “of” means multiply, “is” means equals, and “what” or “a certain” means “x” (or whatever variable you prefer)

Algebra

Linear equations

Finding slope from two points: $\frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$

Standard form of a linear equation: $Ax + By = C$

- The slope of a line in standard form: $-\frac{A}{B}$

Slope-intercept form of a linear equation: $y = mx + b$

$m = slope$

$b = y - intercept$

Parallel lines have identical slopes.

Perpendicular lines have negative reciprocal slopes.

Quadratic equations

Standard form of a quadratic equation: $ax^2 + bx + c = 0$

X-coordinate of the vertex of a parabola in standard form: $-\frac{b}{2a}$

Quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant (for finding the # of solutions): $b^2 - 4ac$

- If greater than zero, 2 solutions
- If equal to zero, 1 solution
- If less than zero, no solutions

Vertex form of a parabola: $y = a(x-h)^2 + k \rightarrow (h,k)$ is the vertex

Difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Perfect square quadratics: $a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

Exponent rules

Operation	Result
Multiplying identical bases: $a^2 \cdot a^3$	Add exponents: a^5
Dividing identical bases: $a^6 \div a^2$	Subtract exponents: a^4
Raising one power to another power: $(a^2)^3$	Multiply exponents: a^6
Multiplying two bases with the same exponent: $a^2 \cdot b^2$	Multiply bases, retain the exponent: $(ab)^2$
Dividing two bases with the same exponent: $a^3 \div b^3$	Divide bases, retain the exponent: $(\frac{a}{b})^3$
Raising a base to the zero power: a^0	Anything to the zero power (except zero) is 1.
Raising a base to a negative power: a^{-2}	The negative exponent acts like a fraction bar, creating a reciprocal: $\frac{1}{a^2}$
Raising a base to a fractional power: $a^{\frac{2}{3}}$	The denominator of the fraction is the root (radical) number; the numerator remains an exponent: $\sqrt[3]{a^2}$: or : $(\sqrt[3]{a})^2$

Miscellaneous

Inequalities

When dividing or multiplying by a negative number, change the direction of the inequality.

Proportions and cross-multiplying

Go From: $\frac{a}{b} = \frac{c}{d}$ to $: ad = bc$

Solving systems of equations

Approach	Execution	Works best when
Substitution	1. Isolate one variable in either equation and plug its equivalent into the other equation. 2. Solve the second equation for its one variable. 3. Plug your answer back into the first equation to solve for the other variable.	One variable is already isolated (or would be easy to isolate) in one of the two equations.
Elimination	1. Multiply one or both equations by a constant so that the equations share an equal but opposite (negative/positive) coefficient for one of the two variables (if the equations already possess an equal but opposite coefficient, proceed to step 2). 2. Add the two equations to eliminate one variable. 3. Solve for the remaining variable in the resulting (sum) equation. 4. Plug the resulting value back into the more accessible of the original two equations and solve for the other variable.	1. The equations already possess an equal but opposite coefficient. 2. One equation can be easily multiplied by a constant so that it has an equal but opposite coefficient to the other equation. 3. Neither substitution nor graphing is easy to execute (note: this means that elimination is often the best way to solve a CLT system of equations).
Graphing	Sketch both equations and estimate the point of intersection.	The equations are already in slope-intercept form, or can be easily manipulated into slope-intercept form.
Backsolving (plugging in answers)	Plug in the answer choices into both equations.	The equations look complicated/intimidating enough that none of the above three approaches appears feasible in a reasonable amount of time.

Geometry

Angles

- When parallel lines are crossed by a non-perpendicular transversal, the following is true:
  - ➩ All acute angles are congruent.
  - ➩ All obtuse angles are congruent.
  - ➩ Any acute angle is supplementary with any obtuse angle.
- Vertical angles (the angles across from each other) are congruent.
- Angles along a straight angle are supplementary.
- There are ***360°*** in a quadrilateral.
- 2π radians equals ***360°*** and π radians equals ***180°***.

Other formulas

- The volume of a rectangular prism is equal to ***length x width x height.***
- The volume of a cube is equal to $edge^3$
- The volume of a cylinder is equal to $\pi r^2 h$
- The equation of a sine function is

  $f(x) = a \sin b(x - c) + d$
  - $a = amplitude$
  - $\frac{2\pi}{b} = period$
  - $c = phase\ shift\ (left/right)$
  - $d = vertical\ shift$

Strategy insight

One way to find the degrees in a polygon is to draw all diagonals from one vertex, then count the triangles formed and multiply by ***180°***

Triangles

- Triangles are proven similar by the AA theorem (two corresponding pairs of angles equal)
- Similar triangles have congruent corresponding angles and proportional corresponding sides
- Congruent triangles can be proved by the following theorems: SSS, SAS, AAS, ASA
- The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$ where *S = Side*
- The most common Pythagorean triples are:
  - ➩ 3:4:5
  - ➩ 5:12:13
  - ➩ 7:24:25
  - ➩ 8:15:17

Circles

- The equation of a circle is $(x–h)^2 + (y - k)^2 = r^2$
- The angle measure of an arc is equal to the measure of the circle's central angle.
- The angle measure of an arc is equal to twice the measure of the inscribed angle.

Strategy insight

- Doubling the radius of a circle ***quadruples*** the area of the circle.
- Doubling the radius of a sphere increases the volume of the sphere by a factor of ***eight.***

Quadrilaterals and other polygons

- The perimeter of a rectangle is equal to $2l + 2w$.
- The area of a square is equal to s^2
- The perimeter of a square is equal to 4s.
- The total degrees in a polygon is determined by $(n - 2)(180)$ where "***s***" is the number of sides.
- The measure of each angle of a regular polygon is equal to $\frac{(n-2)(180)}{n}$

Formulas provided on the CLT

- Area of a circle $= \pi r^2$, where r is the radius of the circle.
- Circumference of a circle $= 2\pi r$ where r is the radius of the circle.
- There are 360° in a circle.
- There are 2π radians in a circle.
- Volume of a sphere $= \frac{4}{3}\pi r^3$, where r is the radius of the sphere.
- Surface area of a sphere $4\pi r^2$, where r is the radius of the sphere.
- Area of a rectangle = *length x width*
- Area of a triangle = $\frac{1}{2}(base\ x\ height)$
- The sum of the measures of the interior angles of a triangle is 180°
- Pythagorean theorem (for a right triangle): If ***a*, *b*,** and ***c*** are the side lengths of the triangle and ***c*** is the hypotenuse, then $a^2 + b^2 = c^2$

Trigonometry continued

$30^\circ - 60^\circ - 90^\circ$ triangles have side lengths in a ratio of $1 : \sqrt{3} : 2$, corresponding to their opposite angle.

$45^\circ - 45^\circ - 90^\circ$ triangles have side lengths in a ratio of $1 : 1 : \sqrt{2}$, corresponding to their opposite angle.

Trigonometry

$$\sin\theta = \frac{opposite}{hypotenuse}$$

$$\cos\theta = \frac{adjacent}{hypotenuse}$$

$$\tan\theta = \frac{opposite}{adjacent}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\cot\theta = \frac{1}{\tan\theta}$$

$$sin^2\theta + \cos^2\theta = 1$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$