Problem solving and data

Measures of central tendency

$Average = sum/number$

Also, importantly:

$Sum = average \times number$

The median is the middle number when the numbers are listed in order. (If there is no middle number, take the middle two numbers and average them to find the median.)

The mode is the number that appears most frequently in a list. There can be multiple modes if there is a "tie".

Percents

$$\% = \frac{part}{whole}$$

$$\%\ change = \frac{(\ new - old\ )}{old}$$

To enter a percent in a calculator, convert it to a decimal first.

*So **50% = 0.5.***

With percents, "***of***" means multiply; "***is***" means equals, and "***what***" or "***a* certain**" means "***x***" (or whatever variable you prefer).

Data figures

A histogram shows the frequency that each data point occurs. When calculating the median of a set of data, the frequency of each value must be considered.

In a scatterplot, the line of best fit shows predicted values while the individual points show actual values.

Range and standard deviation

The range is the difference between the largest and smallest numbers in a list.

The standard deviation is a measure of how spread out the data are. For a large standard deviation, look for numbers that are far spread out from each other. And keep in mind that two data sets can have very different means (averages) while having the same standard deviation, if the data look exactly the same on a dot plot or histogram.

Probability

$Probability = desired\ outcomes/total\ outcomes$

Probability is shown as a fraction or decimal and not as a percent (unless percent is indicated).

A 100% probability, therefore, = **1.**

Dependent probability occurs with multiple selections without replacement. If there are three red balls out of six total balls and you draw two balls out, the probability they are both red is:

$$\frac{3}{6} \times \frac{2}{5} = \frac{1}{5}$$

Algebra

Linear equations

Finding slope from two points: $\frac{rise}{run} = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$

Standard form of a linear equation: $Ax + By = C$

- The slope of a line in Standard Form: $-\frac{A}{B}$

Slope-intercept Form of a linear equation: $y = mx + b$

$m = slope$

$b = y - intercept$

Parallel lines have **identical** slopes.

Perpendicular lines have **negative reciprocal** slopes.

A linear equation may have:

one solution: Ex. $3x + 7 = 12$

no solution: Ex. $3x + 7 = 3x + 12$

infinite solutions: Ex. $3x + 7 = 3x + 7$

Linear inequalities

When solving an inequality, **change the direction of the inequality symbol when dividing or multiplying by a negative.**

With systems of inequalities, plug in points to see which values work.

Graphing inequalities:

Dashed line: less than or greater than

Solid line: less than **or equal to** or greater than **or equal to**

y > (constant or expression); shade **above** the line

y < (constant or expression); shade **below** the line

Exponent rules

Operation		Result	
Multiplying identical bases:	$a^2 \cdot a^3$	Add exponents:	a^5
Dividing identical bases:	$a^6 \div a^2$	Subtract exponents:	a^4
Raising one power to another power:	$(a^2)^3$	Multiply exponents:	a^6
Multiplying two bases with the same exponent:	$a^2 \cdot b^2$	Multiply bases, retain the exponent:	$(ab)^2$
Dividing two bases with the same exponent:	$a^3 \div b^3$	Divide bases, retain the exponent:	$(\frac{a}{b})^3$
Raising a base to the zero power:	a^0	Anything to the zero power (except zero) is 1.	
Raising a base to a negative power:	a^{-2}	The negative exponent acts like a fraction bar, creating a reciprocal:	$\frac{1}{a^2}$
Raising a base to a fractional power:	$a^{\frac{2}{3}}$	The denominator of the fraction is the root (radical) number; the numerator remains an exponent: $\sqrt[3]{a^2}$ *or* $(\sqrt[3]{a})^2$	

Polynomials

In a polynomial such as $ax^3 + bx^2 + cx + d$,

the constant term (d in this case) provides the **y-intercept**.

The **x-intercepts**, also known as the zeroes or roots of the polynomial, are the values of **x** for which the polynomial is equal to zero.

Algebra, continued

Solving systems of equations

Approach	Execution	Works best when
Substitution *As in:* $x = y + 7$ $2x - 3y = 11$	1. Isolate one variable in either equation and plug its equivalent into the other equation. 2. Solve the second equation for its one variable. 3. Plug your answer back into the first equation to solve for the other variable.	One variable is already isolated (or would be easy to isolate) in one of the two equations.
Elimination *As in:* $x - 2y = 6$ $3x + 2y = -14$	1. Multiply one or both equations by a constant so that the equations share an equal but opposite (negative/positive) coefficient for one of the two variables (if the equations already possess an equal but opposite coefficient, proceed to step 2). 2. Add the two equations to eliminate one variable. 3. Solve for the remaining variable in the resulting (sum) equation. 4. Plug the resulting value back into the more accessible of the original two equations and solve for the other variable.	1. The equations already possess an equal but opposite coefficient. 2. One equation can be easily multiplied by a constant so that it has an equal but opposite coefficient to the other equation. 3. Neither substitution nor graphing is easy to execute (note: this means that elimination is often the best way to solve a SAT system of equations).
Graphing	Use Desmos often! Familiarize yourself with its features and practice regularly. For skilled students, it's often the fastest way to solve algebra problems.	If Desmos can be used to solve an SAT math problem, it is almost always the best way to do it!
Backsolving (plugging in answers)	Plug the answer choices into both equations.	The equations look complicated/intimidating enough that none of the above three approaches appears feasible in a reasonable amount of time.

Algebra, continued

Exponential equations

In *exponential form* $y = ab^x$

a represents the initial amount

b represents the multiplier applied at a regular time interval

(for example, if ***b* = *2***, the amount doubles every interval)

If ***b* > *1***, the function represents exponential growth.

If ***b* < *1***, the function represents exponential decay.

The ***y-intercept*** is simply $y = a, since\ b^0 = 1$

Absolute value equations and inequalities

Absolute value equations create two normal equations: one representing the positive value and one the negative value.

Example: $|2x + 7| = 9$

$$2x + 7 = 9 \quad OR \quad 2x + 7 = -9$$

Absolute value inequalities create two normal inequalities: one representing the positive value and one the negative value (with the direction of the symbol reversed).

Example: $|2x + 7| > 9$

$$2x + 7 > 9 \quad OR \quad 2x + 7 < -9$$

Quadratic equations

Standard form of a quadratic equation: $ax^2 + bx + c = 0$

Always set quadratic expressions equal to zero before solving.

X-coordinate of the vertex of a parabola in Standard Form: $-\frac{b}{2a}$

Quadratic formula: $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant (for finding the # of solutions): $b^2 - 4ac$

- If greater than zero, 2 solutions
- If equal to zero, 1 solution
- If less than zero, no solutions

Vertex form of a parabola: $y = a(x - h)^2 + k \rightarrow (h, k)$ is the vertex

Difference of squares: $a^2 - b^2 = (a - b)(a + b)$

Perfect square quadratics: $a^2 + 2ab + b^2 = (a + b)^2$

$a^2 - 2ab + b^2 = (a - b)^2$

The sum of the solutions to a quadratic given in standard form is equal to $\frac{-b}{a}$

The product of the solutions to a quadratic given in standard form is equal to $\frac{c}{a}$

Other formulas

With proportions, go from:

$$\frac{a}{b} = \frac{c}{d}$$

to:

$$ad = bc$$

When solving a radical equation such as $\sqrt{x + 5} = x - 1$ look for an extraneous solution that doesn't actually work. Plug in your answers to check!

The "rate" formula is distance = rate x time.

This formula also applies to "work/rate": if you encounter a question about two people or machines working together, use the formula (with ***"work"*** replacing ***"distance"***) to find their individual rates, then add the rates together.

Geometry

Angles

- When parallel lines are crossed by a non-perpendicular transversal, the following is true:
  - All acute angles are congruent.
  - All obtuse angles are congruent.
  - Any acute angle is supplementary with any obtuse angle.
- Vertical angles (the angles across from each other) are congruent.
- Angles along a straight angle are supplementary.
- There are *360°* in a quadrilateral.
- 2π radians equals *360°* and π radians equals *180°*.

3D solids

- The volume of a cube is equal to $edge^3$.
- The surface area of a cube is equal to $6s^2$ where *s* is side length.
- The surface area of a cylinder is equal to $2\pi r^2 + 2\pi rh$.

Strategy insight

- Doubling the radius of a circle **quadruples** the area of the circle.
- Doubling the radius of a sphere increases the volume of the sphere **by a factor of eight**.

Quadrilaterals and other polygons

- The perimeter of a rectangle is equal to $2l + 2w$ or $2(l+w)$
- The total degrees in a polygon is determined by $(n-2)(180)$, where *n* is the number of sides.
- The measure of each angle of a regular polygon is equal to $\frac{(n-2)(180)}{n}$

Triangles

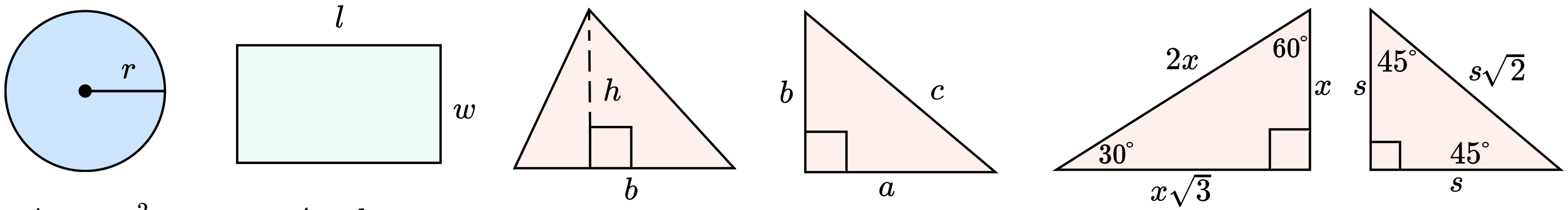
- Triangles are proven similar by the AA theorem (two corresponding pairs of angles equal)
- Similar triangles have congruent corresponding angles and proportional corresponding sides
- Congruent triangles can be proved by the following theorems: SSS, SAS, AAS, ASA
- The area of an equilateral triangle is $\frac{s^2\sqrt{3}}{4}$ where *S=Side*
- The most common Pythagorean triples are:
  - 3:4:5
  - 5:12:13
  - 7:24:25
  - 8:15:17
- A triangle inscribed in a semicircle must be a right triangle.
- Right triangle trigonometry uses SohCahToa:

$$sine = \frac{opposite}{hypotenuse} \qquad cosine = \frac{adjacent}{hypotenuse} \qquad tangent = \frac{opposite}{adjacent}$$

Circles

- The length of the diameter is twice the length of the radius.
- The equation of a circle is $(x-h)^2 + (y-k)^2 = r^2$.
  - (h,k) is the center of the circle and r is the length of the radius.
  - If the circle is not presented in this form, you may need to complete the square to match it in the form above.
- The angle measure of an arc is equal to the measure of the circle's central angle.
- The angle measure of an arc is equal to twice the measure of the inscribed angle.
- Arc length is equal to $\frac{\theta}{360} \times (2\pi r)$, where θ is the measure of the central angle.
- Sector area is equal to $\frac{\theta}{360} \times (\pi r^2)$, where θ is the measure of the central angle.
- A tangent line to a circle is perpendicular to the radius at the point of tangency.

Formulas provided on the SAT



$A = \pi r^2$

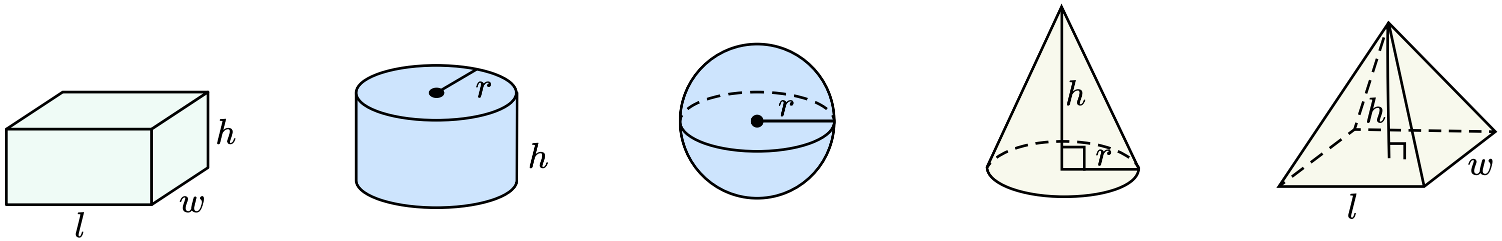
$C = 2\pi r$

$A = lw$

$A = \frac{1}{2}bh$

$c^2 = a^2 + b^2$

Special Right Triangles



$V = lwh$

$V = \pi r^2 h$

$V = \frac{4}{3}\pi r^3$

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}lwh$

The number of degrees of arc in a circle is ***360.***

The number of radians of arc in a circle is ***2π.***

The sum of the measures in degrees of the angles of a triangle is ***180.***