Achievable

AMC 10/12 DUMP SHEET

Identities

Algebra

- Factoring:
  - $a^2 - b^2 = (a - b)(a + b)$
  - $a^2 + 2ab + b^2 = (a + b)^2$
  - $a^2 - 2ab + b^2 = (a - b)^2$
  - $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
  - $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
- Exponents:
  - $x^0 = 1$ for $x \neq 0$
  - $x^y x^z = x^{y+z}$
  - $\dfrac{x^y}{x^z} = x^{y-z}$
  - $x^{-z} = \dfrac{1}{x^z}$
  - $(x^y)^z = x^{yz}$
  - $x^z y^z = (xy)^z$
- Logarithms:
  - $\log(ab) = \log a + \log b$
  - $\log \dfrac{a}{b} = \log a - \log b$
  - $\log(a^n) = n \log a$
- Euler's identity: $e^{i\pi} + 1 = 0$

Trig identities

- $\tan x = \dfrac{\sin x}{\cos x}$
- $\sin^2 \theta + \cos^2 \theta = 1$
- $1 + \tan^2 \theta = \sec^2 \theta$
- $1 + \cot^2 \theta = \csc^2 \theta$
- $\sin 2\theta = 2 \sin \theta \cos \theta$
- $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$
- $\sin^2 \theta = \dfrac{1 - \cos 2\theta}{2}$
- $\cos^2 \theta = \dfrac{1 + \cos 2\theta}{2}$

Number theory

- $\operatorname{lcm}(a, b) \cdot \gcd(a, b) = ab$, for positive integers
- If $a \equiv b \pmod{m}$, then:
  - $a + c \equiv b + c \pmod{m}$
  - $a - c \equiv b - c \pmod{m}$
  - $ac \equiv bc \pmod{m}$

Counting and probability

- $\dbinom{n}{k} = \dbinom{n}{n-k}$
- Pascal's rule: $\dbinom{n}{k} = \dbinom{n-1}{k} + \dbinom{n-1}{k-1}$
- Binomial theorem: $(x + y)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$
- $1 + 2 + \cdots + n = \dfrac{n(n+1)}{2}$
- $1^2 + 2^2 + \cdots + n^2 = \dfrac{n(n+1)(2n+1)}{6}$
- $1^3 + 2^3 + \cdots + n^3 = \left(\dfrac{n(n+1)}{2}\right)^2$