Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 6: Riemann sums and the fundamental theorem

Accumulation of change

- $\int_a^b r(t)\,dt$ is the net change in the amount whose rate is r: area above the axis adds, area below subtracts
- Units: the rate's units times the input's units, such as gallons per minute × minutes
- Net change (displacement) uses the signed areas; a total (distance) adds their sizes

Riemann and trapezoidal sums

- n equal subintervals on $[a, b]$: $\Delta x = \dfrac{b-a}{n}$, $x_k = a + k\Delta x$
- **Left:** $L_n = \Delta x\,[f(x_0) + f(x_1) + \cdots + f(x_{n-1})]$
- **Right:** $R_n = \Delta x\,[f(x_1) + f(x_2) + \cdots + f(x_n)]$
- **Midpoint:** $M_n = \Delta x\,[f(m_1) + \cdots + f(m_n)]$, with $m_k = \dfrac{x_{k-1} + x_k}{2}$
- **Trapezoidal:** $T_n = \dfrac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)]$
- $T_n = \dfrac{L_n + R_n}{2}$, the average of the left and right sums
- **Unequal widths** (a table): give each piece its own width. A trapezoid is $\dfrac{w}{2}\,[f(\text{left}) + f(\text{right})]$

Sum	Overestimates when f is	Underestimates when f is
Left	Decreasing	Increasing
Right	Increasing	Decreasing
Trapezoidal	Concave up	Concave down

The definite integral

- The limit of a Riemann sum: $\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\,\Delta x$, the net signed area
- **Read a limit as an integral:** match $\Delta x = \dfrac{b-a}{n}$ and $x_k = a + k\Delta x$. Shifting a shifts f:

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n}\left(1 + \frac{k}{n}\right)^2 = \int_1^2 x^2\,dx = \int_0^1 (1+x)^2\,dx$$

Fundamental theorem of calculus

- **Part 1,** for f continuous and a a constant: $\dfrac{d}{dx}\int_a^x f(t)\,dt = f(x)$
- Variable limits: $\dfrac{d}{dx}\int_{h(x)}^{g(x)} f(t)\,dt = f(g(x))\,g'(x) - f(h(x))\,h'(x)$. With a constant lower limit, the second term is 0
- **Part 2,** for f continuous on $[a, b]$ and F any antiderivative: $\int_a^b f(x)\,dx = F(b) - F(a)$
- So $F(b) = F(a) + \int_a^b f(x)\,dx$: the starting value plus the net change

Properties

- $\int_a^a f(x)\,dx = 0$
- $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
- $\int_a^c f\,dx = \int_a^b f\,dx + \int_b^c f\,dx$
- $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
- $\int_a^b [f \pm g]\,dx = \int_a^b f\,dx \pm \int_a^b g\,dx$, but there is no such rule for a product or a quotient

Accumulation functions

- For $F(x) = \int_a^x f(t)\,dt$, write $F'(x) = f(x)$ first; then $F''(x) = f'(x)$ and $F(a) = 0$

Where f	F
Is positive (negative)	Increases (decreases)
Changes from + to −	Relative maximum
Changes from − to +	Relative minimum
Increases (decreases)	Concave up (down)
Has a relative extremum	Inflection point

- **Absolute extrema** of F on $[a, b]$: compare F at the endpoints and where $f = 0$
- From a graph of f, find values of F by geometry: triangles, trapezoids, and semicircles of area $\frac{1}{2}\pi r^2$