Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 1: Limits and continuity

Limits

- $\lim_{x \to a} f(x) = L$: as x approaches a, $f(x)$ approaches L. $f(a)$ need not equal L, or even be defined
- The limit exists only if both one-sided limits exist and are equal: $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$
- It does not exist (DNE) when the one-sided limits differ, the function grows without bound, or it oscillates
- **Limit laws:** when both limits exist, a sum, difference, product, constant multiple or power takes the limits' values; so does a quotient, if the bottom's limit is not 0

Evaluating limits

- **Direct substitution first:** polynomial, rational, radical, exponential, log and trig functions have $\lim_{x \to a} f(x) = f(a)$ wherever $f(a)$ is defined
- $\frac{0}{0}$ is indeterminate, not DNE. Simplify, then substitute:
  - Factor and cancel
  - Rationalize: multiply by the conjugate
  - Multiply a complex fraction by the common denominator
  - L'Hôpital's rule (Unit 4)
- **Absolute value or piecewise:** write out the pieces and take each one-sided limit at the break
- **Composite:** $\lim_{x \to a} f(g(x)) = f(L)$ if $g(x) \to L$ and f is continuous at L. If not, on each side of a, take f's one-sided limit from the side g nears L from. If the two differ, it DNE

Special limits

- $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$ and $\lim_{x \to \infty} \frac{\sin x}{x} = 0$
- **Squeeze theorem:** if $g(x) \le f(x) \le h(x)$ near a and $\lim_{x \to a} g(x) = \lim_{x \to a} h(x) = L$, then $\lim_{x \to a} f(x) = L$

Asymptotes

- **Vertical** $x = a$: $f(x) \to \pm\infty$ as $x \to a^-$ or $x \to a^+$. A rational function has one where its simplified form gives $\frac{c}{0}$, $c \neq 0$: each one-sided limit is ∞ or $-\infty$
- **Hole** at $x = a$: substitution gives $\frac{0}{0}$ and, after cancelling, the simplified form is defined at a. Its height is that value
- **Horizontal** $y = L$: $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$. Check both
- A rational function, by the degrees of top and bottom:
  - Top lower: $y = 0$
  - Equal: the ratio of the leading coefficients
  - Top higher: none; the limit is $\pm\infty$
- $\sqrt{x^2} = |x|$, which is $-x$ as $x \to -\infty$
- Growth as $x \to \infty$: $\ln x \ll x^p \ll e^x$, for $p > 0$

Continuity

- f is continuous at $x = a$ if $f(a)$ is defined, $\lim_{x \to a} f(x)$ exists, and the two are equal
- Discontinuities:
  - **Removable** (a hole): the limit exists but $f(a)$ is undefined or different
  - **Jump:** the one-sided limits exist but differ
  - **Infinite:** a vertical asymptote
- **Piecewise:** at a break, set the one-sided limits equal to each other and to $f(a)$ to solve for a constant

Intermediate value theorem (IVT)

- If f is continuous on $[a, b]$ and L is between $f(a)$ and $f(b)$, then $f(c) = L$ for at least one c between a and b
- **Justify** by stating both: f is continuous on $[a, b]$, and $f(a) < L < f(b)$ (or $f(a) > L > f(b)$)
- From a table: each interval whose ends straddle L holds at least one solution

The exam, from May 2027

- **Multiple choice:** 42 questions, 1 hour 40 minutes, 50% of the score
  - Part A: 29 questions, 62 minutes, no calculator
  - Part B: 13 questions, 38 minutes, graphing calculator
- **Free response:** 6 questions, 1 hour 30 minutes, 50%
  - Part A: 2 questions, 30 minutes, graphing calculator
  - Part B: 4 questions, 60 minutes, no calculator
  - Calculator answers to three decimal places

Unit (multiple-choice weight)	AB	BC
1 Limits and continuity	10—15%	5—10%
2 Definition and basic rules	10—15%	5—10%
3 Composite, implicit, inverse	5—10%	5—10%
4 Contextual applications	10—15%	5—10%
5 Analytical applications	15—20%	10—15%

Unit (multiple-choice weight)	AB	BC
6 Integration and accumulation	15—20%	15—20%
7 Differential equations	5—10%	5—10%
8 Applications of integration	10—15%	5—10%
9 Parametric, polar, vector	—	10—15%
10 Sequences and series	—	15—20%