Achievable

AP CALCULUS BC DUMP SHEET

Units 6—8: The BC-only topics

Integration by parts

- $\int u\,dv = uv - \int v\,du$: it reverses the product rule.
  Definite: $\int_a^b u\,dv = \Big[uv\Big]_a^b - \int_a^b v\,du$
- Choose u by **LIATE**, the first type present: logarithmic, inverse trig, algebraic, trig, exponential. The rest is dv
- $\int \ln x\,dx$: take $dv = dx$, giving $x\ln x - x + C$
- x^n times e^x or a trig function: repeat n times.
  $\int x^2e^x\,dx = x^2e^x - 2xe^x + 2e^x + C$
- $e^x\sin x$ or $e^x\cos x$: apply it twice, with the same type as u both times, until the integral returns; then solve for it

Partial fractions

- The fraction must be proper (top degree lower); if not, use long division first
- Factor the bottom into distinct linear factors:
  $\dfrac{P(x)}{(x-a)(x-b)} = \dfrac{A}{x-a} + \dfrac{B}{x-b}$
- Multiply through by the bottom, then set $x = a$ and $x = b$ to find A and B
- Each piece integrates to a log:
  $\int \dfrac{A}{x-a}\,dx = A\ln|x-a| + C$
- $\dfrac{3x-37}{x^2-3x-4} = \dfrac{8}{x+1} - \dfrac{5}{x-4}$, so its integral is
  $8\ln|x+1| - 5\ln|x-4| + C$

Improper integrals

- An infinite bound, or an integrand unbounded on the interval (an asymptote at an end or inside)
- Replace it with a limit: $\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$.
  Split at an inside asymptote, with one-sided limits
- Both bounds infinite: split at any c into $\int_{-\infty}^c$ and $\int_c^\infty$, each its own limit; never one symmetric limit $\int_{-b}^b$
- Converges if the limit is a finite number; diverges otherwise. If any piece diverges, the whole integral does
- $\int_1^\infty \dfrac{dx}{x^2} = 1$ converges; $\int_1^\infty \dfrac{dx}{x}$ diverges

Euler's method

- Step along tangent lines from the initial point, with step h:
  $x_{\text{new}} = x + h$, $y_{\text{new}} = y + \dfrac{dy}{dx}\Big|_{(x,y)} \cdot h$
- $h = \dfrac{\text{target } x - \text{start } x}{\text{number of steps}}$, negative to step left
- Keep a table of x, y, the slope and the new y, and carry unrounded values
- One step is a tangent-line step: if the solution is concave up across it ($\dfrac{d^2y}{dx^2} > 0$), the estimate is an **underestimate**; concave down, an **overestimate**

Logistic growth

- $\dfrac{dP}{dt} = kP\left(1 - \dfrac{P}{L}\right) = \dfrac{k}{L}P(L-P)$: growth jointly proportional to P and $L - P$
- **Carrying capacity** L: the nonzero solution of $\dfrac{dP}{dt} = 0$. For $P(0) > 0$, $\lim\limits_{t\to\infty} P(t) = L$
- $0 < P < L$: P increases. $P > L$: it decreases toward L
- **Fastest growth** at $P = \frac{L}{2}$: concave up for $0 < P < \frac{L}{2}$, concave down for $\frac{L}{2} < P < L$, and concave up again above L. Starting below $\frac{L}{2}$, the curve is an S with its inflection point there
- **Solution,** with k from $kP\left(1 - \dfrac{P}{L}\right)$: $P(t) = \dfrac{L}{1 + Ce^{-kt}}$, $C = \dfrac{L - P(0)}{P(0)}$. From $\dfrac{dP}{dt} = cP(L-P)$, use $k = cL$

Arc length

- For f' continuous on $[a, b]$: $L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$
- For $x = g(y)$ on $[c, d]$: $L = \int_c^d \sqrt{1 + [g'(y)]^2}\,dy$
- Parametric arc length is on the Unit 9 sheet