Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 8: Applications of integration

Average value

- For f continuous on $[a, b]$: $f_{\text{avg}} = \dfrac{1}{b-a}\displaystyle\int_a^b f(x)\,dx$, in the units of f
- **Average value of a rate** $R(t)$: $\dfrac{1}{b-a}\displaystyle\int_a^b R(t)\,dt$. **Average rate of change** of an amount $V(t)$: $\dfrac{V(b)-V(a)}{b-a}$. The two are equal when $V' = R$

Accumulation in context

- **Net change:** $\displaystyle\int_a^b R(t)\,dt = V(b) - V(a)$, so $V(t) = V(0) + \displaystyle\int_0^t R(s)\,ds$
- The amount increases where $R > 0$, since $V' = R$. The rate increases where $R' > 0$

Motion

- **Displacement** $= \displaystyle\int_a^b v(t)\,dt$, the net signed area under v
- **Total distance** $= \displaystyle\int_a^b |v(t)|\,dt$, the total area. By hand, split where v changes sign
- **Position:** $x(b) = x(a) + \displaystyle\int_a^b v(t)\,dt$. Likewise $v(b) = v(a) + \displaystyle\int_a^b a(t)\,dt$

Area between curves

- **Vertical slices:** $A = \displaystyle\int_a^b [\text{top} - \text{bottom}]\,dx$
- **Horizontal slices:** $A = \displaystyle\int_c^d [\text{right} - \text{left}]\,dy$, with x in terms of y
- Find the bounds where the curves meet. Split wherever they cross, or on a calculator integrate $|f(x) - g(x)|$
- Use dy when top and bottom change partway but right and left do not
- An area is never negative

Volume by cross sections

- Perpendicular to the x-axis: $V = \displaystyle\int_a^b A(x)\,dx$, with side $s = \text{top} - \text{bottom}$
- Perpendicular to the y-axis: $V = \displaystyle\int_c^d A(y)\,dy$, with $s = \text{right} - \text{left}$

Cross section on base s	Area
Square	s^2
Rectangle of height h	sh
Equilateral triangle	$\frac{\sqrt{3}}{4}s^2$
Isosceles right triangle, leg on the base	$\frac{1}{2}s^2$
Isosceles right triangle, hypotenuse on the base	$\frac{1}{4}s^2$
Semicircle, diameter s	$\frac{\pi}{8}s^2$
Quarter circle, radius s	$\frac{\pi}{4}s^2$

Volume of revolution

- **Disk** (an edge of the region lies along the axis, so no gap): $V = \pi\displaystyle\int_a^b [R(x)]^2\,dx$
- **Washer** (a gap between region and axis): $V = \pi\displaystyle\int_a^b \left([R(x)]^2 - [r(x)]^2\right)dx$
  Never $\pi(R-r)^2$
- For a region on one side of the axis, R runs from the axis to the farther curve, r to the nearer
- A horizontal axis ($y = k$) takes dx and vertical radii; a vertical axis ($x = k$) takes dy, with x in terms of y
- About $y = k$, a radius is the distance to the line: $|f(x) - k|$