Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 5: Analytical applications of differentiation

Existence theorems

- **Mean value theorem:** if f is continuous on $[a, b]$ and differentiable on (a, b), then $f'(c) = \dfrac{f(b) - f(a)}{b - a}$ for at least one c in (a, b): somewhere the tangent is parallel to the secant
- **Rolle's theorem:** the special case $f(a) = f(b)$, which gives $f'(c) = 0$ for some c in (a, b)
- **Extreme value theorem:** if f is continuous on $[a, b]$, it has at least one absolute maximum and one absolute minimum there, each at a critical point or an endpoint
- Check the hypotheses before using a theorem. If one fails, the theorem says nothing either way

Increasing, decreasing, extrema

- **Critical point:** $x = c$ with $f'(c) = 0$ or $f'(c)$ undefined, and $f(c)$ defined
- $f' > 0$: f increasing. $f' < 0$: f decreasing
- **First derivative test** at a critical point c:
  - f' changes from $+$ to $-$: relative maximum
  - f' changes from $-$ to $+$: relative minimum
  - No sign change: neither
- **Second derivative test,** where $f'(c) = 0$:
  - $f''(c) > 0$: relative minimum
  - $f''(c) < 0$: relative maximum
  - $f''(c) = 0$: inconclusive, so use the first
- Mark where the function is undefined (vertical asymptotes, holes) on both sign charts: those points split the intervals but are never extrema or inflection points
- **Candidates test** for absolute extrema of a continuous f on $[a, b]$: evaluate f at every critical point and both endpoints. The largest value is the maximum, the smallest the minimum
- Justify with f': "f has a relative maximum at $x = 1$ because f' changes from positive to negative there"

Concavity and inflection

- $f'' > 0$: concave up, and f' increasing. $f'' < 0$: concave down, f' decreasing
- **Inflection point:** a point on the graph ($f(c)$ defined) where f changes concavity, so f'' changes sign. $f''(c) = 0$ or undefined only makes c a candidate

Reading the graph of f'

On the graph of f'	What f does
Above the axis	Increases
Below the axis	Decreases
Crosses from above to below	Relative maximum
Crosses from below to above	Relative minimum
Rising	Concave up
Falling	Concave down
A peak or a valley	Inflection point

Optimization

- Write the quantity to maximize or minimize; use the constraint to put it in one variable
- Set its derivative to 0, then justify with either derivative test. On a closed interval, compare the endpoints too
- Only one critical point on an interval where the function is continuous: its relative extremum is absolute
- Motion: farthest right maximizes $x(t)$; fastest maximizes $|v(t)|$; velocity decreasing fastest minimizes $a(t)$