Achievable

Glossary of formulas and theorems: Algebra, number theory, counting, and probability

AMC 10/12 DUMP SHEET

Algebra

Quadratic formula

When $ax^2 + bx + c = 0$ $\quad x = \dfrac{-b \pm \sqrt{(b^2) - 4ac}}{2a}$

Vieta's formula for quadratics
(r represent the roots of the equation)

- $ax^2 + bx + c = 0$
- $r_1 + r_2 = -\dfrac{b}{a}, \quad r_1 r_2 = \dfrac{c}{a}$

Vieta's formula for cubic polynomials

- $P(x) = ax^3 + bx^2 + cx + d$
- $r_1 + r_2 + r_3 = -\dfrac{b}{a}$
- $r_1 r_2 r_3 = -\dfrac{d}{a}$

Vieta's formula for higher degree polynomials

- $P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$
- $r_1 + r_2 + \ldots + r_{n-1} + r_n = -\dfrac{a_{n-1}}{a_n}$
- $r_1 r_2 \ldots r_n = (-1)^n \dfrac{a_0}{a_n}$

Newton sums

- $f(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n$
- $P_1 + a_1 = 0$
  $P_2 + a_1 P_1 + 2a_2 = 0$
  $P_3 + a_1 P_2 + a_2 P_1 + 3a_3 = 0$
  $P_4 + a_1 P_3 + a_2 P_2 + a_3 P_1 + 4a_4 = 0$

Work-Rate Formula

- Work = Rate (Time)

Euler's formula

- $e^{i\theta} = \cos\theta + i \sin\theta$

Number theory

Fermat's Little Theorem

- $a^{p-1} = 1 \pmod{p}$

Euler's theorem (*a* and *n* must be coprime and $\varphi(n)$ is the number of positive integers less than *n* that do not share any prime factors with *n*)

- $a^{\Phi(n)} = 1 \pmod{n}$

Sum of squares

- $\displaystyle\sum_{n=1}^{n} a^2 = 1^2 + 2^2 + 3^2 + \ldots\ldots n^2 = \frac{n(n+1)(2n+1)}{6}$

Sum of cubes

- $\displaystyle\sum_{n=1}^{n} n^3 = 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$

Counting and probability

- Permutations = $\dfrac{n!}{(n-k)!}$
- Combinations = $\dfrac{n!}{k!(n-k)!}$

Binomial theorem (used in both algebra and combinatorics problems)

- $(a+b)^n = \displaystyle\sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$