Trig identities, area, and volume

Area between curves

- Vertical slices (top minus bottom):
  - $A = \int_a^b [f(x) - g(x)]\,dx$
- Horizontal slices (right minus left):
  - $A = \int_c^d [f(y) - g(y)]\,dy$

Volume

- **Cross-sections:** $V = \int_a^b A(x)\,dx$, where $A(x)$ is the cross-section's area
- **Disk method:** $V = \pi \int_a^b [R(x)]^2\,dx$
- **Washer method:** $V = \pi \int_a^b \left([R(x)]^2 - [r(x)]^2\right) dx$
- For a vertical axis of rotation, integrate in dy, with A, R and r in terms of y

Trig identities

- Reciprocal and quotient identities:
  - $\sin\theta = \dfrac{1}{\csc\theta}$
  - $\csc\theta = \dfrac{1}{\sin\theta}$
  - $\cos\theta = \dfrac{1}{\sec\theta}$
  - $\sec\theta = \dfrac{1}{\cos\theta}$
  - $\tan\theta = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$
  - $\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$
- Pythagorean identities:
  - $\sin^2\theta + \cos^2\theta = 1$
  - $\tan^2\theta + 1 = \sec^2\theta$
  - $1 + \cot^2\theta = \csc^2\theta$
- Double angle formulas:
  - $\sin 2\theta = 2\sin\theta\cos\theta$
  - $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- Sum and difference formulas:
  - $\sin(a \pm b) = \sin a\cos b \pm \cos a \sin b$
  - $\cos(a \pm b) = \cos a \cos b \mp \sin a \sin b$
  - $\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$
- Half angle (power-reducing) formulas:
  - $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$
  - $\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$