Theorems

AP Calc AB dump sheet

Important theorems

- **Intermediate value theorem**
  - If a function f is continuous on a closed interval $[a, b]$, then it will take on every function value between $f(a)$ and $f(b)$.
- **Extreme value theorem**
  - If f is continuous on a closed interval $[a, b]$, then it must have an absolute max and an absolute min.
  - To find absolute extrema, compare function values at critical points and endpoints.
- **Mean value theorem**

  If

  1. f is continuous on the closed interval $[a, b]$
  2. Differentiable on the open interval (a, b)

  Then there must be a c in (a, b) where

  $$f'(c) = \frac{f(b) - f(a)}{b - a}$$

- **Rolle's theorem**
  - Special case of the Mean value theorem where $f'(c) = 0$

  If

  1. f is continuous on the closed interval $[a, b]$
  2. Differentiable on the open interval (a, b)
  3. $f(a) = f(b)$

  Then there must be a c in (a, b) where $f'(c) = 0$.

Important theorems

- $x = c$ is a critical point if $f'(c) = 0$ or $f'(c)$ is undefined and changes sign.
  - $f(x)$ must be continuous at c
- **1st derivative test for extrema**

  Analyze the sign changes of $f'(x)$ around the critical point $x = c$
  - $f(c)$ is a relative maximum if $f'(x)$ changes from positive to negative
  - $f(c)$ is a relative maximum if $f'(x)$ changes from negative to positive
- **2nd derivative test for extrema**

  Analyze the sign of f'' at the critical point $x = c$
  - If $f''(c) > 0$, then $f(c)$ is a relative minimum
  - If $f''(c) < 0$, then $f(c)$ is a relative maximum
  - If $f''(c) = 0$, the test is inconclusive
- **Point of inflection**

  If $f''(c) = 0$ or is undefined and changes sign at $x = c$, then $(c, f(c))$ is an inflection point (f must be continuous at $x = c$).

Important theorems

- Use the tangent line at $x = a$

  $L(x) = f'(a)(x - a) + f(a)$

  $L(x)$ is the approximate of the actual value $f(x)$ for values of x near a.
- To determine if $L(x)$ overestimates or underestimates the actual value, analyze the concavity.
  - If $f''(a) > 0$ (concave up), the tangent line **underestimates.**
  - If $f''(a) < 0$ (concave down), the tangent line overestimates.
  - If $f''(c) = 0$, the test is inconclusive