Achievable

Riemann sum approximations
AP Calc AB dump sheet

Riemann sum approximations

For n equal subintervals on $[a,b]$:

$$\Delta x = \frac{b-a}{n}$$

Left Riemann sum:

$$L_n = \Delta x\,[f(x_0) + f(x_1) + \cdots + f(x_{n-1})]$$

- Uses left endpoints
- Excludes rightmost endpoint

Right Riemann sum:

$$R_n = \Delta x\,[f(x_1) + f(x_2) + \cdots + f(x_n)]$$

- Uses right endpoints
- Excludes leftmost endpoint

Midpoint Riemann sum:

$$M_n = \Delta x\,[f(m_1) + f(m_2) + \cdots + f(m_n)]$$

- Where $m_k = \frac{x_{k-1} + x_k}{2}$

Trapezoidal rule:

$$T_n = \frac{\Delta x}{2}\,[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + f(x_n)]$$

Limit definition of the definite integral:

$$\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{k=1} \Delta x \cdot f(x_k)$$

- Where $x_k = a + i\Delta x$