Achievable

AP CALCULUS AB DUMP SHEET

Limits

Limits

- A limit exists if and only if both one-sided limits exist and are equal
- If $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$, then $\lim_{x \to a} f(x) = L$

Continuity

- f is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$: it is defined there and equals the limit

Differentiability and continuity

- If f is **differentiable** at $x = a$, it is **continuous** at $x = a$
- A function can be continuous but not differentiable at $x = a$: a corner, a cusp, or a vertical tangent

Limits at infinity (horizontal asymptotes)

- For a rational function $h(x) = \frac{f(x)}{g(x)}$, with f and g polynomials:
  - Degree of f < degree of g: $\lim_{x \to \infty} h(x) = 0$
  - Degree of f > degree of g: $\lim_{x \to \infty} h(x) = \pm\infty$, no horizontal asymptote
  - Equal degrees: the limit is the ratio of the leading coefficients

Finding limits algebraically

- Techniques:
  - Direct substitution
  - Factor and cancel
  - Simplify rational expressions
  - Rationalize with conjugates (for radicals)
  - Multiply by a common denominator
  - Use trig identities when trig is involved

Special limits

- $\lim_{x \to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$
- $\lim_{x \to \infty} \frac{\sin(ax)}{bx} = 0$
- $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$
- $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$

L'Hôpital's rule

- If direct substitution gives $\frac{0}{0}$ or $\frac{\infty}{\infty}$, differentiate the top and bottom separately and re-evaluate: $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

Indeterminate forms

- Rewrite or simplify with the techniques above; for $\frac{0}{0}$ and $\frac{\infty}{\infty}$, use L'Hôpital's rule:
  - $\frac{0}{0}$
  - $\frac{\infty}{\infty}$
  - $0 \cdot \infty$
  - $\infty - \infty$
  - 1^∞
  - ∞^0
  - 0^0