# Limits

## AP Calc AB dump sheet

### Limits

**A limit exists if and only if both one-sided limits exist and equal each other.**

- if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$
- then $\lim_{x\to a} f(x) = L$

### Continuity

- A function $f(x)$ is continuous at a point $x = a$ if $\lim_{x\to a} f(x) = f(a)$ (function is defined and equals the limit).

### Limits

- If a function is **differentiable** at $x = a$, then it is **continuous** at $x = a$.
- However, a function can be continuous but not differentiable if $x = a$ is a corner, cusp, or has a vertical tangent.

### Limits at infinity (horizontal asymptotes)

- Let $h(x)$ be a rational function $\frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are polynomials

1. If the degree of $f(x) < g(x)$, then $\lim_{x\to\infty} h(x) = 0$
2. If the degree of $f(x) < g(x)$, then $\lim_{x\to\infty} h(x) = \infty$
3. If the degree of $f(x) = g(x)$, then $\lim_{x\to\infty} h(x) =$ ratio of the leading coefficients

### Finding limits algebraically

- **Limit techniques:**
  - Direct substitution
  - Factor and cancel
  - Simplify rational expressions
  - Rationalize using conjugates (for radicals)
  - Multiply by a common denominator
  - Use trig identities if trig is involved

### Special limits

- $\lim_{x\to 0} \frac{\sin(ax)}{bx} = \frac{a}{b}$
- $\lim_{x\to 0} \frac{\sin(ax)}{bx} = 0$
- $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$
- $\lim_{x\to \infty} \left(1 + \frac{1}{x}\right)^x = e$

### L'Hopital's rule

- If direct substitution results in $\frac{0}{0}$ or $\frac{\infty}{\infty}$ differentiate the top and bottom and reevaluate:

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

### Special limits

- If direct substitution results in these limits, use the above techniques to rewrite or simplify the limit.

For $\frac{0}{0}$ and $\frac{\infty}{\infty}$, use L'Hopital's rule.

- $\frac{0}{0}$
- $\frac{\infty}{\infty}$
- $0 \times \infty$
- $\infty - \infty$
- $1^\infty$
- $\infty^0$
- $0^0$