Achievable

AP CALCULUS AB DUMP SHEET

Integration and particle motion

Integration

- **Reverse power rule**, for most polynomial integrals:
  $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$, for $n \neq -1$
- For other functions, use the derivative tables in reverse. For example:
  - To integrate $\int \frac{1}{x} \, dx$, ask which function has this derivative
  - From the tables, $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$
  - So $\int \frac{1}{x} \, dx = \ln|x| + C$

u-substitution

- Reverses the chain rule
- Set u to a part of the integrand whose derivative du appears, in some form, elsewhere in the integral

Particle motion

- Position, velocity, acceleration:
  - Position: $s(t)$
  - Velocity: $v(t) = s'(t)$
  - Acceleration: $a(t) = v'(t) = s''(t)$
- Speed increases when $v(t)$ and $a(t)$ have the same sign, and decreases when their signs are opposite
- **Displacement** from $t = a$ to $t = b$: $\int_a^b v(t) \, dt = s(b) - s(a)$
- **Total distance** from $t = a$ to $t = b$: integrate speed, $\int_a^b |v(t)| \, dt$

Average value

- $f_{\text{avg}} = \frac{1}{b-a} \int_a^b f(x) \, dx$
- f_{avg} has the same units as $f(x)$

Fundamental theorem of calculus

- Part 1: derivatives and integrals are inverse operations
  - $\frac{d}{dx} \int_a^x f(t) \, dt = f(x)$
  - With an upper limit $g(x)$, add the chain rule:
    $\frac{d}{dx} \int_a^{g(x)} f(t) \, dt = f(g(x)) \, g'(x)$
- Part 2: if f is continuous on $[a, b]$ and F is any antiderivative of f
  - $\int_a^b f(x) \, dx = F(b) - F(a)$