Integration and particle motion
AP Calc AB dump sheet

Achievable

Integration

- The reverse power rule handles most polynomial integrals:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

For other functions, use the derivative tables in reverse.

Example:

To integrate $\int \frac{1}{x}\, dx$:

1. Which function has this derivative?
2. From the derivative tables, $\frac{d}{dx}[ln|x|] = \frac{1}{x}$
3. Reverse it, so $\int \frac{1}{x}\, dx = ln|x| + C$

u-substitution

- Reverses the chain rule
- Set u to be a portion of the integrand such that its derivative du appears in some form in another part of the integral.

Fundamental theorem of calculus

- **Part 1:**

Derivatives and integrals are inverse operations

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

If the upper limit is a function $g(x)$, then the FTC combined with the chain rule states that

$$\frac{d}{dx}\int_a^{g(x)} f(t)dt = f(g(x))g'(x)$$

- **Part 2:**

If f is continuous on $[a, b]$, and F is any antiderivative of f, then to compute a definite integral, integrate and evaluate at the endpoints:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Particle motion

Position $= s(t)$
Velocity $= v(t) = s'(t)$
Acceleration $= a(t) = v'(t) = s''(t)$

A particle accelerates (its speed increases) when $v(t)$ and a(t) have the same sign.

It decelerates (its speed decreases) when the signs are opposite.

The displacement from time $t = a$ to $t = b$ is

$$\int_a^b v(t)dt = s(b) - s(a)$$

The total distance traveled from time $t = a$ to $t = b$ is found by integrating speed, or the absolute value of velocity

$$\int_a^b |v(t)|dt$$

Average value

- The average value of a function $f(x)$ is

$$f_{avg} = \frac{1}{b\text{-}a}\int_a^b f(x)dx$$

The units of f_{avg} are the same as those of $f(x)$