Achievable

AP CALCULUS AB DUMP SHEET

Important theorems

Existence theorems

- Intermediate value theorem:
  - If f is continuous on $[a, b]$, it takes every value between $f(a)$ and $f(b)$
- Extreme value theorem:
  - If f is continuous on $[a, b]$, it has an absolute maximum and an absolute minimum there
  - To find them, compare the function's values at the critical points and the endpoints
- Mean value theorem:
  - If f is continuous on $[a, b]$ and differentiable on (a, b), there is a c in (a, b) with $f'(c) = \frac{f(b) - f(a)}{b - a}$
- Rolle's theorem:
  - The mean value theorem's special case: if also $f(a) = f(b)$, there is a c in (a, b) with $f'(c) = 0$

Critical points and extrema

- **Critical point:** $x = c$, in the domain of f, where $f'(c) = 0$ or $f'(c)$ is undefined
- **First derivative test** at a critical point $x = c$:
  - f' changes from positive to negative: $f(c)$ is a relative maximum
  - f' changes from negative to positive: $f(c)$ is a relative minimum
- **Second derivative test** where $f'(c) = 0$:
  - $f''(c) > 0$: relative minimum
  - $f''(c) < 0$: relative maximum
  - $f''(c) = 0$: inconclusive
- **Point of inflection:** if $f''(c) = 0$ or is undefined and f'' changes sign at $x = c$, with f continuous there, $(c, f(c))$ is an inflection point

Linear approximation

- The tangent line at $x = a$, $L(x) = f'(a)(x - a) + f(a)$, approximates $f(x)$ for x near a
- Over- or underestimate? Check the concavity:
  - $f''(a) > 0$ (concave up): the tangent line **underestimates**
  - $f''(a) < 0$ (concave down): it **overestimates**
  - $f''(a) = 0$: inconclusive