Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 7: Differential equations

Differential equations

- An equation relating an unknown function to its derivatives
- **Proportional to y:** $\frac{dy}{dt} = ky$. **Inversely proportional:** $\frac{dy}{dt} = \frac{k}{y}$. Find k by substituting given values
- To check a solution, differentiate it and substitute into the equation: both sides must match
- There may be infinitely many solutions: the **general** solution has $+C$; an initial condition picks the **particular** one

Slope fields

- At each point (x, y), a short segment of slope $\frac{dy}{dx}$. A solution curve follows the segments
- Horizontal segments where $\frac{dy}{dx} = 0$; where it is undefined, no segment (or a vertical one)
- Same slope down each column: $\frac{dy}{dx}$ depends only on x. Along each row: only on y
- Match a field to an equation by the sign of the slope in each quadrant
- Sketch a particular solution through the given point, along the segments

Using the equation directly

- **Slope at a point:** substitute it into $\frac{dy}{dx}$
- **Tangent line** at the initial condition (x_0, y_0): $y = y_0 + m(x - x_0)$, where m is $\frac{dy}{dx}$ at (x_0, y_0)
- **Concavity:** differentiate $\frac{dy}{dx}$ implicitly, then substitute $\frac{dy}{dx}$ to get $\frac{d^2y}{dx^2}$

Separation of variables

- For $\frac{dy}{dx} = f(x)\,g(y)$: separate to $\frac{dy}{g(y)} = f(x)\,dx$. If it will not factor that way, the method does not apply. Where $g(y) = 0$, y is a constant solution that dividing loses: check it first
- Steps:
  - Separate the variables
  - Integrate both sides; add $+C$ on the side of x
  - Substitute the initial condition straight away to find C
  - Solve for y, choosing the sign of a $\pm$ or an absolute value from the initial condition
- $\frac{dy}{dx} = 2xy$ gives $\ln|y| = x^2 + C$, so $y = Ae^{x^2}$, with A a new constant ($A = 0$ is the solution $y = 0$)

Exponential models

- $\frac{dy}{dt} = ky$ has the solution $y = y_0e^{kt}$, where $y_0 = y(0)$. It may be stated without separating, unless the question says to separate
- $k > 0$: growth. $k < 0$: decay
- **Doubling time** T: $k = \frac{\ln 2}{T}$. **Half-life** T: $k = -\frac{\ln 2}{T}$
- A rate proportional to a difference, $\frac{dH}{dt} = k(M - H)$, is not this form: separate the variables and show every step