Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 2: The derivative and its basic rules

The derivative

- **Average rate of change** on $[a, b]$, the secant line's slope: $\frac{f(b) - f(a)}{b - a}$. No limit needed
- **Instantaneous rate of change**, the tangent line's slope: $f'(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h} = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$
- **The derivative function:** $f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$, the limit of the difference quotient. Also written $\frac{dy}{dx}$ or $\frac{d}{dx}[f(x)]$
- **A limit that is a derivative:** $\lim_{h \to 0} \frac{(3 + h)^2 - 9}{h} = f'(3)$ for $f(x) = x^2$, so it is 6
- **From a table,** estimate $f'(a)$ by the average rate of change over the closest values on either side of a, written out: $f'(3.5) \approx \frac{f(4) - f(3)}{4 - 3}$. A bare number loses the point. Units: output units per input unit

Basic rules

Rule	Derivative
Constant c	$\frac{d}{dx}[c] = 0$
Power (any real n)	$\frac{d}{dx}[x^n] = nx^{n-1}$
Constant multiple	$\frac{d}{dx}[cf(x)] = cf'(x)$
Sum and difference	$[f \pm g]' = f' \pm g'$
Product	$[fg]' = fg' + f'g$
Quotient	$\left[\frac{f}{g}\right]' = \frac{gf' - fg'}{g^2}$

- Rewrite roots and quotients as powers first: $\frac{2}{5x^3} = \frac{2}{5}x^{-3}$, $\sqrt[4]{x^3} = x^{3/4}$
- The power rule needs a constant exponent: $\frac{d}{dx}[2^x] \neq x \cdot 2^{x-1}$

Tangent and normal lines

- **Tangent** at $(a, f(a))$: $y - f(a) = f'(a)(x - a)$
- **Normal:** perpendicular to the tangent, slope $-\frac{1}{f'(a)}$. If $f'(a) = 0$, it is the vertical line $x = a$
- **Horizontal tangent:** solve $f'(x) = 0$. Parallel to a line: set $f'(x)$ equal to its slope

Trig, exponential and log derivatives

- $\frac{d}{dx}[\sin x] = \cos x$
- $\frac{d}{dx}[\cos x] = -\sin x$
- $\frac{d}{dx}[\tan x] = \sec^2 x$
- $\frac{d}{dx}[\cot x] = -\csc^2 x$
- $\frac{d}{dx}[\sec x] = \sec x \tan x$
- $\frac{d}{dx}[\csc x] = -\csc x \cot x$

With $b > 0$ and $b \neq 1$:

- $\frac{d}{dx}[e^x] = e^x$
- $\frac{d}{dx}[\ln x] = \frac{1}{x}$
- $\frac{d}{dx}[b^x] = b^x \ln b$
- $\frac{d}{dx}[\log_b x] = \frac{1}{x \ln b}$

Differentiability and continuity

- Differentiable at $x = a$ means continuous there. Not continuous means not differentiable. Continuous does not mean differentiable: $|x|$ at 0
- Not differentiable at:
  - A discontinuity
  - A corner, where the one-sided slopes differ: $|x|$ at 0
  - A cusp, where they go to opposite infinities: $x^{2/3}$ at 0
  - A vertical tangent: $x^{1/3}$ at 0
- A piecewise function is differentiable at a break if the pieces meet (continuity) and their derivatives are equal there