Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 4: Contextual applications of differentiation

Derivatives in context

- Units of f': output units per input unit. Units of f'': those of f' per input unit
- Interpret a derivative with all four parts: when, what, increasing or decreasing, and how fast, with units
  - "At $t = 5$ minutes, the volume of water is increasing at a rate of 12 gallons per minute"
- Say increasing or decreasing; never "changing at a rate of negative 3"
- A negative derivative of a rate means the rate is decreasing. If the rate is still positive, the quantity is still increasing, only more slowly

Straight-line motion

- Velocity $v(t) = x'(t)$; acceleration $a(t) = v'(t) = x''(t)$; speed $= |v(t)|$
- $v > 0$: moving right (or up). $v < 0$: left. $v = 0$: at rest
- **Changes direction** only where v changes sign; $v = 0$ alone is not enough
- **Speeding up** when v and a have the same sign; **slowing down** when they differ
- Average velocity $\dfrac{x(t_2) - x(t_1)}{t_2 - t_1}$; average acceleration $\dfrac{v(t_2) - v(t_1)}{t_2 - t_1}$
- **Displacement** $x(b) - x(a)$. **Total distance:** split where v changes sign and add each piece's distance
- **On a velocity graph:** the slope is a, the height above or below the axis is the speed, and a crossing of the axis is a change of direction
- **From a table:** v is continuous and changes sign between two times, so by the IVT $v = 0$ between them

Related rates

- Steps:
  - Draw it; name the variables and the rates given and wanted
  - Write an equation relating the variables
  - Differentiate both sides with respect to t
  - Substitute the values, then solve for the unknown rate
- Substitute a changing quantity only after differentiating; a constant may go in before
- A quantity increasing has a positive rate; decreasing, negative
- Useful: $x^2 + y^2 = z^2$, similar triangles, sphere $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$, cone $V = \frac{1}{3}\pi r^2 h$
- Angles in radians

Linear approximation

- The tangent line at $x = a$, $L(x) = f(a) + f'(a)(x - a)$, approximates $f(x)$ for x near a
- If $f'' > 0$ between a and x (concave up), it **underestimates**; if $f'' < 0$, it **overestimates**

L'Hôpital's rule

- If $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} g(x) = 0$, or both are $\pm\infty$, then $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \lim\limits_{x \to a} \dfrac{f'(x)}{g'(x)}$, if the new limit exists or is $\pm\infty$
- Differentiate the top and bottom separately, not with the quotient rule. Repeat if needed
- On a free response, show both limits first. Never write $\lim \dfrac{f}{g} = \frac{0}{0}$