Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 3: Composite, implicit, and inverse functions

Chain rule

- $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$: the outside's derivative, inside left alone, times the inside's derivative
- In Leibniz form: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
- With $u = g(x)$:
  - $\frac{d}{dx}[u^n] = nu^{n-1}u'$
  - $\frac{d}{dx}[e^u] = e^u u'$
  - $\frac{d}{dx}[\ln u] = \frac{u'}{u}$
  - $\frac{d}{dx}[\sin u] = \cos u \cdot u'$
- **From a table:** $h(x) = f(g(x))$ gives $h'(a) = f'(g(a))\,g'(a)$
- Decide the outermost structure first (product, quotient or composition), then work inward

Implicit differentiation

- Differentiate both sides with respect to x. Each y term picks up a factor $\frac{dy}{dx}$: $\frac{d}{dx}[y^2] = 2y\frac{dy}{dx}$, $\frac{d}{dx}[xy] = x\frac{dy}{dx} + y$
- Then solve for $\frac{dy}{dx}$. For a slope at a point, substitute the point right after differentiating, before solving
- **Horizontal tangent:** the top of $\frac{dy}{dx}$ is 0 and the bottom is not. **Vertical:** the bottom is 0 and the top is not. $\frac{0}{0}$ needs more work
- **Second derivative:** differentiate $\frac{dy}{dx}$ again, substitute $\frac{dy}{dx}$, and simplify with the original equation. $x^2 + y^2 = 25$ gives $\frac{dy}{dx} = -\frac{x}{y}$ and $\frac{d^2y}{dx^2} = -\frac{25}{y^3}$

Inverse functions

- If $g = f^{-1}$, then $g'(a) = \frac{1}{f'(g(a))}$, where $f'(g(a)) \neq 0$
- Find $g(a) = b$ by solving $f(b) = a$, then $g'(a) = \frac{1}{f'(b)}$
- The slope of f^{-1} at (a, b) is the reciprocal of the slope of f at (b, a)
- $f(g(x)) = x$ in a problem signals inverse functions

Logarithmic differentiation

- Take $\ln$ of both sides ($\ln|y|$ if y can be negative), expand with the log rules, differentiate ($\frac{d}{dx}[\ln|y|] = \frac{y'}{y}$), then multiply by y
- $\ln(ab) = \ln a + \ln b$, $\ln\frac{a}{b} = \ln a - \ln b$, $\ln a^b = b\ln a$

Higher-order derivatives

- $f''(x) = \frac{d^2y}{dx^2}$, then $f'''(x)$, $f^{(4)}(x)$, ...
- The derivatives of $\sin x$ and $\cos x$ repeat every four

Inverse trig derivatives

- $\frac{d}{dx}[\sin^{-1} x] = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}[\cos^{-1} x] = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$
- $\frac{d}{dx}[\cot^{-1} x] = -\frac{1}{1+x^2}$
- $\frac{d}{dx}[\sec^{-1} x] = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}[\csc^{-1} x] = -\frac{1}{|x|\sqrt{x^2-1}}$
- Each co-function's derivative is the negative of its partner's
- For an inner function u, replace x with u and multiply by u': $\frac{d}{dx}[\tan^{-1} u] = \frac{u'}{1+u^2}$
- $\sin^{-1} x$ is $\arcsin x$, not $\frac{1}{\sin x}$