Achievable

AP CALCULUS AB / BC DUMP SHEET

Unit 6: Antiderivatives and integration techniques

Basic antiderivatives

Integrand	Antiderivative
$x^n,\ n \neq -1$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln\lvert x\rvert + C$
e^x	$e^x + C$
a^x	$\frac{a^x}{\ln a} + C$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
$\sec^2 x$	$\tan x + C$

Integrand	Antiderivative
$\csc^2 x$	$-\cot x + C$
$\sec x \tan x$	$\sec x + C$
$\csc x \cot x$	$-\csc x + C$
$\frac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + C$
$\frac{1}{1+x^2}$	$\tan^{-1} x + C$
$\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$	$\sec^{-1} x + C$

- Every indefinite integral ends in $+C$, unless an initial condition fixes it: integrate, then substitute the condition to find C
- Rewrite first: expand, split a fraction over its denominator, write roots as powers. $\int \frac{1-2\sin x}{\cos^2 x}\,dx = \int (\sec^2 x - 2\sec x \tan x)\,dx$

u-substitution

- Reverses the chain rule. Use it when an inner function's derivative (or a constant multiple of it) is also there
- Let $u = g(x)$, so $du = g'(x)\,dx$. Rewrite everything in u, integrate, then put $g(x)$ back
- **Definite integral:** change the bounds to $u = g(a)$ and $u = g(b)$. Never mix u with the x-bounds
- A leftover x: solve $u = g(x)$ for x and substitute
- $\int 2x\cos(x^2)\,dx = \sin(x^2) + C$

Trig identities

- $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$, $1 + \cot^2\theta = \csc^2\theta$
- $\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
- $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$, $\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$

Long division

- When the top's degree is at least the bottom's, divide first, then integrate the polynomial and the proper fraction
- $\frac{2x^2+3x+5}{x+1} = 2x + 1 + \frac{4}{x+1}$, so its integral is $x^2 + x + 4\ln\lvert x+1\rvert + C$

Completing the square

- For a quadratic that does not factor: $x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$. With $-x^2$, factor out the minus first: $8 - 2x - x^2 = 9 - (x+1)^2$
- Then, with $a > 0$ and du from the substitution (for $u = 3x$, $dx = \frac{1}{3}\,du$):
  - $\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$
  - $\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$
- $x^2 + 4x + 6 = (x+2)^2 + 2$, so $\int \frac{2\,dx}{x^2+4x+6} = \sqrt{2}\tan^{-1}\frac{x+2}{\sqrt{2}} + C$