Achievable

GRE DUMP SHEET

# Geometry

## Angles and lines

- **Vertical (opposite) angles** are equal; the parts of an angle add up to the whole
- A straight line is $180°$; all the way around a point, $360°$
- **Parallel lines and a transversal:** all acute angles equal, all obtuse angles equal, acute + obtuse = 180°
- **Parallel:** same slope. **Perpendicular:** $90°$, slopes $\frac{2}{3}$ and $-\frac{3}{2}$
- Fill in every angle you can before hunting for the one asked

## Triangles

- **Area** $\frac{1}{2}bh$: the height is perpendicular to the base (a leg, in a right triangle); angles sum to $180°$
- **Equilateral:** all sides equal, each angle 60°. **Isosceles:** at least two equal sides, with equal angles opposite them
- **Right:** the hypotenuse, opposite the 90° angle, is the longest side. **Scalene:** nothing equal
- **Third side rule:** $a - b < c < a + b$; sides 4 and 9: $5 < x < 13$, and the edges are traps

## Figures: what to trust

- **Geometric figures are not necessarily drawn to scale:** never take a length, an angle, a right angle or parallel lines from the look of one
- **Do assume:** lines shown straight are straight, points are in the order shown, shapes sit where they're shown
- **Drawn to scale:** xy-planes, number lines and data graphs

## Right triangles

- **Pythagorean theorem** $a^2 + b^2 = c^2$: right triangles only, $c$ the hypotenuse
- A missing leg is $\sqrt{c^2 - a^2}$, but look for a triple before you compute
- **Triples** and their multiples: $3:4:5$, $5:12:13$, $7:24:25$
- $45°$-$45°$-$90°$: $x : x : x\sqrt{2}$; any isosceles right triangle, half a square (diagonal $s\sqrt{2}$)
- $30°$-$60°$-$90°$: $x : x\sqrt{3} : 2x$; half an equilateral triangle, the hypotenuse twice the short leg

## Quadrilaterals and polygons

| Shape | What makes it | Area | Perimeter |
|---|---|---|---|
| Rectangle | Four right angles; opposite sides parallel and equal | $bh$ | $2b + 2h$ |
| Square | A rectangle with four equal sides | $s^2$, or $\frac{d^2}{2}$ from the diagonal | $4s$ |
| Rhombus | Four equal sides; opposite angles equal | $bh$, or $\frac{pq}{2}$ from the diagonals | $4s$ |
| Parallelogram | Opposite sides parallel and equal | $bh$ | the sides |
| Trapezoid | At least one pair of parallel sides, the bases | $\frac{1}{2}(b_1 + b_2)h$ | the sides |

- **Interior angles** of an $n$-sided polygon sum to $(n - 2) \times 180°$: 540° (5 sides), 720° (6), 1080° (8). **Exterior angle** = 180° − interior
- **Regular** (all sides and angles equal): each angle is the sum $\div n$: 108° (5), 120° (6), 135° (8). A regular hexagon is six equilateral triangles

## Circles

- **Area** $\pi r^2$; **circumference** $2\pi r = \pi d$
- **Arc** $\frac{\theta}{360} \cdot 2\pi r$; **sector** $\frac{\theta}{360} \cdot \pi r^2$ ($\theta$ the central angle)
- An arc measures the same as its **central angle**; a circle is 360°
- **Central angle theorem:** a central angle is twice an inscribed angle on the same arc
- A right triangle inscribed in a circle has the diameter as its hypotenuse
- A tangent is perpendicular to the radius where they meet
- Square in a circle: diagonal = diameter. Circle in a square: diameter = side
- Answers in $\pi$? Keep $\pi$; don't round along the way

## 3D shapes

| Solid | Volume | Surface area |
|---|---|---|
| Cube | $s^3$ | $6s^2$ |
| Rectangular solid | $lwh$ | $2lw + 2lh + 2wh$ |
| Cylinder | $\pi r^2 h$ | $2\pi r^2 + 2\pi rh$ |

- Volume = **base area × height**; a cube's diagonal is $s\sqrt{3}$

## Shaded regions

- **Whole minus what sits on it,** or the sum of the shaded pieces
- Square of perimeter 16 minus a circle of diameter 2: $16 - \pi$
- **Inscribed:** fits exactly inside. **Circumscribed:** fits exactly around