Achievable

GRE DUMP SHEET

Algebra and coordinate geometry

Solving equations

- Undo each step on **both** sides: constants first, coefficient last
- $3x - 3 = 27 \to 3x = 30 \to x = 10$; $x^2 = 100 \to x = \pm 10$
- $[0, 10]$ includes the ends; $(0, 10)$ excludes them

Absolute value and inequalities

- $|x|$: distance from 0, **never negative**; $|-3| = 3$
- On a number line, a to b is $|a - b|$; midway is their average
- **Isolate the bars, solve twice:** $2|3 + x| = 8 \to x = 1$ or -7
- $|x - 5| \geq 3 \to x \geq 8$ **or** $x \leq 2$; $|A| < B \to -B < A < B$
- **× or ÷ by a negative: flip the sign:** $-2x > 4 \to x < -2$
- Never × or ÷ both sides by an unknown: it may be negative

Systems of equations

- **Isolate and substitute:** $x = 12 - 3y$ into the other equation
- **Add or subtract** to cancel a variable, every term
- **Scale first:** $\times 2$ turns $-5y$ into $-10y$ to cancel $+10y$
- Asked for $3x + y$? **Build it:** $(6x - 7y = 50) - (3x - 8y = 25)$
- To solve for every variable: as many equations as variables
- Cancels to $0 = 0$? Same line: infinitely many solutions
- To $0 = 5$? Parallel lines: no solution

FOIL and quadratics

- **FOIL:** $(x + 3)(x - 2) = x^2 + x - 6$
- $(a \pm b)^2 = a^2 \pm 2ab + b^2$; $(a + b)(a - b) = a^2 - b^2$
- **Factor:** product c, sum b: $x^2 + 5x - 6 = (x + 6)(x - 1)$
- Set to 0, each factor gives a root: $x = -6, 1$; FOIL to check
- **Leading coefficient:** $2x^2 - 7x - 15 = (2x + 3)(x - 5)$
- **Common factor first:** $4x^2 + 8x - 12 = 4(x^2 + 2x - 3)$
- $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, when factoring fails

Functions

- $f(x) = 4x + 2$: $f(3) = 14$; replace **every** x
- **Nested**, inside first: $g(x) = 3x$, so $f(g(3)) = f(9) = 38$
- $f(x, y) = 2x - y \to f(2, 1) = 3$; $x\star = x - 2.5$ is $f(x)$
- $f(x) = \frac{x-2}{x-1}$ is defined for all x but 1

Word problems

Words	Math	Words	Math
is	$=$	twice a	$2a$
more than	$+$	half of a	$\frac{a}{2}$
less than	$-$	x percent of a	$\frac{x}{100}a$

- 12 more in X than in Y: $X = Y + 12$; T twice S: $T = 2S$
- **Together** $\frac{AB}{A+B}$ (A, B alone); $d = rt$; 0.4 h = 24 min
- **Variables in the choices?** Pick numbers and test each

Lines and the coordinate plane

- **Slope** $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$, the same between any two points
- **Slope-intercept** $y = mx + b$: $4x = -2y + 6$ is $y = -2x + 3$
- **Intercepts:** set $x = 0$ for the y-intercept, $y = 0$ for the x
- **Horizontal** $y = 5$: slope 0; **vertical** $x = -3$: undefined
- **Parallel:** equal slopes; **perpendicular:** $\frac{2}{3}$ and $-\frac{3}{2}$
- **Midpoint** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
- **Distance** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, the Pythagorean theorem
- **Quadrants:** I $(+, +)$, II $(-, +)$, III $(-, -)$, IV $(+, -)$

Parabolas

- $y = ax^2 + bx + c$: **up** if $a > 0$, **down** if $a < 0$
- c is the y-intercept; the roots are the x-intercepts
- **Vertex** $x = -\frac{b}{2a}$; $y = a(x - h)^2 + k$ has vertex (h, k)
- **Meets a line** where equal: $6x + 9 = x^2 + 3x + 5$ at $x = -1, 4$

Graphing inequalities

- $y >$ or $\geq$: shade **above**; $<$ or $\leq$: **below**
- **Dashed** for $<$, $>$; **solid** for $\leq$, $\geq$; two at once: the **overlap**

Circles

- $(x - h)^2 + (y - k)^2 = r^2$, center (h, k)
- $(x + 5)^2 + (y - 4)^2 = 9$: center $(-5, 4)$, $r = 3$
- The center is the midpoint of any diameter
- **Tangent** $\perp$ radius: r = the center-to-line distance
- **Point to line** $Ax + By + C = 0$: $\dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$
- **Gap between separate circles:** center distance $- r_1 - r_2$