Achievable

ACT DUMP SHEET

Pre-algebra and elementary algebra

Order of operations: PEMDAS

- Parentheses, Exponents, then Multiply and Divide left to right, then Add and Subtract left to right
- Work from the **inside out**; a fraction bar and absolute value bars group like parentheses

Absolute value

- Distance from 0, never negative: $|x-5| = -3$ has **no solution**
- Isolate, then split: $4 + |x-12| = 12 \to x - 12 = \pm 8 \to x = 20$ or 4
- Its graph is a **V**, opening up or down

Fractions

- Numerator 0: the fraction is **0**; denominator 0: **undefined**
- **Divide:** times the reciprocal, $\frac{1}{2} \div \frac{3}{4} = \frac{1}{2} \cdot \frac{4}{3}$; **add:** common denominator, add tops
- Cancel **factors** only: $\frac{2(x-1)}{2(y+4)} = \frac{x-1}{y+4}$; $\frac{2x-1}{2y+4}$ keeps its 2s
- Split a sum on top, $\frac{2x+7}{2x} = 1 + \frac{7}{2x}$; **never** below: $\frac{a}{b+c} \neq \frac{a}{b} + \frac{a}{c}$

Percents, ratios and proportions

- **Percent** is per hundred: 40% of 200 is $0.40 \times 200 = 80$
- **Ratio** $3:7$: scale both sides alike, $6:14$; a fraction or percent is **part ÷ whole**
- **Similar figures:** lengths $\times k$ make area $\times k^2$

Scientific notation

- One nonzero digit before the point: $12{,}345{,}678 \approx 1.2 \times 10^7$, $0.000437 = 4.37 \times 10^{-4}$
- **Multiply:** numbers, then **add** exponents, then fix: $21.4 \times 10^{11} = 2.14 \times 10^{12}$
- **Divide:** subtract exponents. **Add:** convert to standard form. Calculator: EE means ×10^

Probability

- $P = \frac{\text{wanted}}{\text{total}}$; several events **all** happening: **multiply**, never add
- **Without replacement:** black then white $\frac{2}{5} \cdot \frac{3}{4} = \frac{3}{10}$

Unit conversions

- 12 in = 1 ft, 3 ft = 1 yd; **milli-** a thousandth (millennium), **centi-** a hundredth (century), **kilo-** 1,000
- Units cancel: $3{,}000{,}000\ \text{mm} \cdot \frac{1\ \text{m}}{1{,}000\ \text{mm}} \cdot \frac{1\ \text{km}}{1{,}000\ \text{m}} = 3\ \text{km}$
- **Area: square the factor:** $1\ \text{ft}^2 = 144\ \text{in}^2$, $1\ \text{yd}^2 = 9\ \text{ft}^2$

Types of numbers

Set	Holds
Natural	1, 2, 3, ...
Whole	0, 1, 2, ...
Integer	..., −1, 0, 1, ...
Rational	$\frac{a}{b}$: ends or repeats

Set	Holds
Irrational	$\sqrt{2}$, π: no pattern
Real	both of those
Imaginary	$3i$, $\sqrt{-16} = 4i$
Complex	$a + bi$: all

- **GCF** of 16 and 48 is 16; **LCM** of 12 and 9 is 36: list them, or factor trees

Exponent rules

Rule	Example
Multiply same bases: **add** exponents	$2x^2 \cdot 3x^3 = 6x^5$
Divide same bases: **subtract**	$\frac{4x^2}{2x} = 2x$
Power of a power: **multiply**	$(2^{3x})^{2y} = 2^{6xy}$
Power of a product: every factor	$(3x)^2 = 9x^2$
Negative: a reciprocal	$2^{-4} = \frac{1}{16}$
Zero power, $a \neq 0$	$a^0 = 1$
A sum: FOIL it	$(x+2)^2 = x^2 + 4x + 4$

Simplifying and isolating

- **Like terms** only (same variables and powers): $x^3 + 2x^3 = 3x^3$; $x^3 + x^2$ stays
- **FOIL:** $(x+3)(x-5) = x^2 - 2x - 15$; **factor out:** $2x^2 + x = x(2x+1)$
- **Isolate** in reverse PEMDAS, same on both sides: $4x + 2 = 10 \to x = 2$
- $f(3)$: every x becomes 3; $g(f(2))$: inside first

Linear equations

- **Slope** $m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$; linear: no power above 1
- $y = mx + b$; **point-slope** $y - y_1 = m(x - x_1)$
- $Ax + By = C$: slope $-\frac{A}{B}$, y-intercept $\frac{C}{B}$; an intercept sets the **other** variable to 0

Key words and sequences

- **+** more, sum; **−** less, difference; **×** times, each; **÷** out of; **=** is
- "Three **less than** four times y" is $4y - 3$: the order flips
- **Arithmetic** $(+d)$: $a_n = a_1 + (n-1)d$, sum $\frac{n}{2}\left(2a_1 + (n-1)d\right)$
- **Geometric** $(\times r)$: $a_n = a_1 r^{n-1}$, sum $a_1 \frac{1-r^n}{1-r}$