Achievable

ACT DUMP SHEET

Intermediate algebra

Modeling word problems

- Note the **numbers**, the **unknown** and the **key words**; draw it
- First month free, then $9 a month, $39 to spend: $9(m-1)=39$, $m \approx 5.3$: **5** months
- Floor left by 30 round tables ($r=1$) in a 20 × 10 room: $200-30\pi \approx 105.8$

Logarithms

- $\log_b a = c$ means $b^c = a$: $\log_2 8 = 3$; $\log_x 9 = 2 \to x = 3$
- Same base throughout, both ways:
  - $\log_b(xy) = \log_b x + \log_b y$
  - $\log_b \frac{x}{y} = \log_b x - \log_b y$
  - $\log_b x^n = n \log_b x$
- **Change of base:** $\log_b a = \frac{\log a}{\log b}$: $\log_2 30 \approx 4.9$
- **Exponent unknown?** Take a log: $33 = 2^x \to x = \log_2 33$
- $\ln = \log_e$, $e \approx 2.718$, $\ln e = 1$: $5e^{3x} = 40 \to x = \frac{\ln 8}{3}$

Roots and rational exponents

- $x^{\frac{m}{n}} = \sqrt[n]{x^m}$: the denominator is the **index**; $x^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{x^2}}$
- **Rationalize:** $\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
- **Prime factors** by factor tree: $12 = 2^2 \cdot 3$, $48 = 2^4 \cdot 3$

Systems of equations

- **Substitute:** $x + y = 7 \to x = 7 - y$, into the other equation
- **Eliminate:** make one variable's coefficients opposites, then add
- Ends true ($3 = 3$): **infinitely many**; ends false ($0 = -5$): **none**

Counting

- Choices in a row: **multiply**: 6 cars, 4 drivers: 24 pairs
- **Factorial** $n! = n(n-1)\cdots 2 \cdot 1$: $5! = 120$
- **Order matters:** ${}_nP_r = \frac{n!}{(n-r)!}$: 4 of 5 guests seated: $5 \cdot 4 \cdot 3 \cdot 2 = 120$
- **Order doesn't:** ${}_nC_r = \frac{n!}{r!\,(n-r)!}$: 2 of 5: 10

Matrices

- **Rows × columns**; AB needs A's columns = B's rows: $(2 \times 3)(3 \times 2)$ is 2×2
- Each entry: a **row** times a **column**, pairs multiplied and added
- $AB \neq BA$; a scalar multiplies every entry

Factoring

- $x^2 + bx + c$: **product** c, **sum** b: $x^2 + 2x - 8 = (x-2)(x+4)$
- $a^2 - b^2 = (a+b)(a-b)$
- $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$
- **Cubic**, usually given a factor: synthetic division by its root; remainder 0 leaves a quadratic

Solving quadratics

- **Factor**, then each factor = 0: $(x+4)(x-1) = 0 \to x = -4, 1$
- $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; a missing term is 0, keep signs
- $b^2 - 4ac$: > 0 two real, $= 0$ one, < 0 two imaginary
- Real roots are the x-intercepts

Complex numbers

- $i = \sqrt{-1}$, $i^2 = -1$; $i, -1, -i, 1$ repeat: i^n by n's remainder ÷ 4
- **Add:** real with real, i with i: $(2+3i) - (4-i) = -2 + 4i$
- **Multiply:** FOIL, then $i^2 = -1$: $(2+3i)(4-i) = 11 + 10i$
- **Divide:** times the **conjugate** over itself: $\frac{2+3i}{1-i} \cdot \frac{1+i}{1+i} = \frac{-1+5i}{2}$

Percent change and expected value

- $\%\ \text{change} = \frac{\text{new}-\text{old}}{\text{old}} \times 100$; up 50%: $\times 1.5$, down 30%: $\times 0.7$
- \$7 to \$17.50: 250% **of** the old wage, a **150% increase**
- $E(X) = x_1P_1 + x_2P_2 + \cdots$, P as decimals adding to 1; it falls between the smallest and largest x

Summary statistics

- **Mean** = sum ÷ count, so **sum = mean × count**
- **Median:** middle of the ordered list (even: mean of the two)
- **Mode:** most frequent; **range:** largest − smallest
- **Standard deviation:** spread from the mean; compared, not computed

ACT Math: the section

- **45 questions** (41 scored), **50 minutes**, **4 choices** each; **no formula sheet**: memorize
- **Calculator:** a permitted 4-function, scientific or graphing one, never computer algebra; Desmos online
- **Pace:** Q1—15 ≤ 30 s, Q16—35 ~1 min, Q36—45 ~1½ min; less sure? Make Q1—40 count
- Annotate, find what is asked, read the choices, cross out, guess and move on
- **No penalty:** answer all 45. "Any positive number a"? Plug in one