Achievable

ACT DUMP SHEET

Coordinate geometry and functions

Lines and coordinates

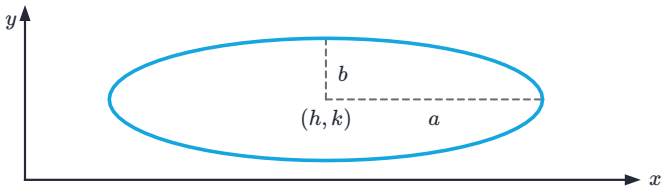
- **Quadrants** I to IV run counterclockwise from top right: $(+,+)$, $(-,+)$, $(-,-)$, $(+,-)$
- **Slope** $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{y_2 - y_1}{x_2 - x_1}$; horizontal: 0; vertical: undefined
- **Slope-intercept** $y = mx + b$; **point-slope** $y - y_1 = m(x - x_1)$
- **Parallel:** same slope; **perpendicular:** $-\frac{1}{m}$, the opposite reciprocal
- **Midpoint** $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$: average each coordinate
- **Distance** $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$: Pythagoras on the grid
- 1 mile west, 2 north, 1 west: $(0,0)$ to $(-2,2)$, $\sqrt{8} \approx 2.8$ miles

Graphing and solving linear equations

- Two points make a line: $y = x + 2$ at $x = 0, 1$: $(0,2)$, $(1,3)$
- **Slope as moves:** from $(1,1)$, slope $\frac{1}{2}$ is up 1, right 2: $(3,2)$
- **x-intercept:** set $y = 0$, solving $mx + b = 0$; **y-intercept:** $x = 0$
- **Two lines:** set equal, $3x - 2 = 4 + x$, for the crossing $(3,7)$
- Cross: one solution; parallel: none; same line: infinitely many

Conic sections

- **Circle:** $(x-h)^2 + (y-k)^2 = r^2$, center (h,k); at the origin $x^2 + y^2 = r^2$
- Right side r^2: $(x-4)^2 + (y+3)^2 = 36$: center $(4,-3)$, $r = 6$
- **Ellipse:** $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, center (h,k)
- a across, b up: $a > b$ wide, $a < b$ tall, $a = b$ a circle
- **Parabola:** $y = a(x-h)^2 + k$, vertex (h,k)



Vectors

- **Components:** end minus start, i for x and j for y: $(5,2)$ to $(7,7)$ is $2i + 5j$, or $\langle 2, 5 \rangle$
- **Add or subtract** like terms: $(-i + 2j) - (2i + 2j) = -3i$
- **Drawn:** head to tail, then start to finish; to subtract, flip the second and add
- **Magnitude** (length) of $ai + bj$: $\sqrt{a^2 + b^2}$; $3i + 4j$ has length 5

Domain and range

- **Domain:** every possible input x; **range:** every possible output y
- **Listed** values go in braces: $\{1, 2, 3, 4, 5\}$, never $[1, 5]$
- $[a, b]$ **includes** its ends (closed dot); (a, b) doesn't (open dot), nor does ∞
- $\cup$ joins intervals: $\frac{1}{x}$ has domain $(-\infty, 0) \cup (0, \infty)$
- **What can't x be?** A denominator isn't 0; even roots need ≥ 0; logs > 0
- **On a graph:** domain left to right, range bottom to top; arrows go on forever

Parent functions

$y =$	Shape	Domain	Range
x^2	U, vertex at the origin	$(-\infty, \infty)$	$[0, \infty)$
$\lvert x \rvert$	V, vertex at the origin	$(-\infty, \infty)$	$[0, \infty)$
x^3	S, rising through the origin	$(-\infty, \infty)$	$(-\infty, \infty)$
$\sqrt{x}$	half a sideways U, from $(0,0)$	$[0, \infty)$	$[0, \infty)$
e^x	through $(0,1)$; asymptote $y = 0$	$(-\infty, \infty)$	$(0, \infty)$
$\log x$	through $(1,0)$; asymptote $x = 0$	$(0, \infty)$	$(-\infty, \infty)$
$\frac{1}{x}$	two branches; both axes asymptotes	$x \neq 0$	$y \neq 0$

Transformations and vertex form

Change	Graph	Change	Graph
$f(x) + k$	up k	$f(x + k)$	left k
$f(x) - k$	down k	$f(x - k)$	right k

- Outside moves it **up or down**; inside moves it **opposite the sign**; $-f(x)$ flips it over the x-axis
- **Vertex form** $a(x-h)^2 + k$ or $a|x-h| + k$: vertex (h,k)
  - $|a| > 1$: narrower; $0 < |a| < 1$: wider; $a < 0$: opens down
  - $y = 2(x+3)^2 + 4$: stretched by 2, **left** 3, up 4
- **From a graph:** vertex for h and k, one more point for a: vertex $(-2,-3)$ through $(0,1)$ is $y = (x+2)^2 - 3$

Asymptotes

- **Vertical:** where a denominator is 0; $\frac{x+3}{(x-1)(x-2)}$ has $x = 1$ and $x = 2$; never crossed
- A factor that **cancels** with the top leaves a hole, not an asymptote
- **Horizontal:** compare the top's and bottom's highest powers; a graph **may** cross it
- Bottom higher: $y = 0$. Equal: the coefficients' ratio, $\frac{x^3 - x}{3x^3 + 4} \to y = \frac{1}{3}$. Top higher: none